## RHEOLOGICAL PROPERTIES OF MATERIALS FROM THE POINT OF VIEW OF PHYSICAL KINETICS

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Zhurkov and Kauzmann rheological bodies, reflecting the physical regularities of plastic deformation of materials, are studied. Solutions of differential equations of flow for composite rheological bodies are obtained. Examples of using structural models of materials that consist of new rheological bodies and describe the inelastic behavior of alloys for various types of temperature-force loading are given. The obtained solutions of the differential equations are used for analysis of the creep of a structurally unstable alloy.

In reproducing the deformation properties of structural materials in calculations, the ideological foundation of concepts of a solid body is very important. Most of the methods used are based on approaches of the mechanics of a solid deformable body, which do not study the causes of deformation processes and do not reflect adequately the entire set of inelastic properties of the material. Only in particular cases of loading using special (often artificial) approaches can admissible solutions be obtained [1-3]. The observed deformation characteristics of solid bodies are caused by a number of internal thermodynamic processes. Modeling of these processes opens up new opportunities in analysis and reflection of macrocharacteristics of materials. Therefore, rheological models of materials, based on the laws of physical kinetics, are more promising and universal and can predict correctly the course of thermodynamic processes when it is difficult to obtain experimental data or they are insufficient.

This approach is based on the theory of absolute reaction rates [4], evolved in 1935. Among the problems studied theoretically, diffusion [5] is closest to strength problems. Kauzmann [6] was one of the first to apply the theory of reaction rates to the flow of solid bodies, treating creep as directed diffusion under the action of applied stresses. Assuming that an applied stress decreases the energetic barrier in one direction and increases it by about the same amount in the opposite direction, he obtained a relation for the excess number of transition acts per unit time in the direction of the applied stress:

$$n^* = 2A \exp\left(-\frac{E}{RT}\right) \sinh\left(\frac{\Delta E}{RT}\right),\tag{1}$$

where E is the initial height of the energy barrier and  $\Delta E$  is the change in this height under the action of the stresses. At large  $\Delta E$ , the return flow through the barrier is usually ignored and relation (1) takes the form

$$n^* = A \exp\left(-\frac{E - \Delta E}{RT}\right). \tag{2}$$

Kauzmann assumed that the quantity  $\Delta E$  depends linearly on the stresses, and relation (2) has been supported by some experimental results.

Since the 1950s fracture and deformation processes of materials have began to be studied in terms of kinetic concepts of the nature of strength [7, 8]. As a result, ideas evolved that provided new insight into all the observed regularities in the behavior of bodies under load [9-11]. Kinetic concepts are the basis in constructing physical models of alloys that describe thermally activated deformation and fracture processes

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and the attendant internal-stress relaxation and structural transformations [12]. This is possible because, in mechanical models of materials, the behavior of viscous elements is described by equations of physical kinetics whose parameters are functions of the structure of the material. Integrally reflecting the real thermodynamic processes in a solid body, these models allow one to predict its behavior over a very wide range of loading conditions, to eliminate a number of conditionalities that still dominate in engineering practice, and to obtain new information on the properties of a specific alloy [13].

**Rheological Models of Solid Bodies.** Besides the classical bodies known in rheology (a Hooke body H, a Newton body N, and a St. Venant body StV [14]), we introduce new bodies. A body whose deformation is described by an expression of the form (2) is called a Zhurkov body Zh [13], and a more complex variant with a flow equation of the type (1) is called a Kauzmann body Km.

Experiments show that for steady flow of a solid body, the plastic-strain rate depends on the stresses and temperature as follows:

$$\frac{d\varepsilon}{dt} = \varepsilon_* \nu_0 \exp\left(-\frac{Q_0 - \alpha\sigma}{RT}\right). \tag{3}$$

Here  $Q_0$  and  $\alpha$  are the activation parameters and  $\varepsilon_*\nu_0$  is the product of the residual strain by the frequency factor [10, 11, 13]. In terms of thermodynamics, relation (3) is of the form [11, 15]

$$\frac{d\varepsilon}{dt} = N^* \varepsilon_0^* \nu_0 \exp\left(-\frac{\Delta G}{kT}\right) = N^* \varepsilon_0^* \nu_0 \exp\left(\frac{\Delta S}{k}\right) \exp\left(-\frac{Q_0 - \alpha\sigma}{kT}\right),\tag{4}$$

where  $\Delta G = \Delta H - T\Delta S$  — the Gibbs free energy — is the driving force of the reaction, and the activation enthalpy  $\Delta H = Q_0 - \alpha \sigma$  is represented by the effective values of the initial activation energy  $Q_0$  and the activation volume  $\alpha$ . The linear dependence of  $\Delta H$  on the stress  $\sigma$  corresponds to numerous experimental data [10], and departure from it is mostly caused by changes in the parameter  $\alpha$ , which is a structurally sensitive coefficient. The preexponent in (4) has the following physical meaning:  $N^*$  is the number of "flow units,"  $\varepsilon_0^*$  is the contribution of each flow unit to the overall deformation of the body,  $\nu_0 \simeq 10^{13} \text{ sec}^{-1}$  is the effective frequency of attempts of atoms to overcome the energy barrier, and  $\Delta S$  is the activation entropy. However, this concept of the kinetic process requires detailing. Rheological models are intended for this.

Denoting  $A = \varepsilon_* \nu_0 \exp(-Q_0/RT)$  and  $B = \alpha/RT$  in (3), we write the rheological equation of a Zh body:

$$d\varepsilon/dt = A \exp{(B\sigma)}.$$
(5)

At low stresses, relation (5) gives deviations from experimental data. Account for the probability of reverse passage of activated complexes through the energy barrier is a physically substantiated correction. If in direct and reverse passage through the barrier, the contributions of deformation processes are assumed to be equal, the rheological equation of state for a Km body

$$d\varepsilon/dt = 2A\sinh\left(B\sigma\right) \tag{6}$$

is more rigorous. Rheological bodies with constant values of the parameters A and B are called ideal bodies.

We form composite bodies of Zh or Km and H bodies. They will be called physical media and be denoted by the symbol PM. In accordance with the notation adopted in rheology [14], the rheological formulas of the basic composite bodies are written as

$PM_1 = H$	-	Zh,					$PM_5 = H$	—	Km,				
$PM_2 = H$	1	Zh,					$PM_6 = H$	I	Km,				
$PM_3 = H$	1	$PM_1 = H$		(H	—	Zh),	$PM_7 = H$	1	$PM_5 = H$		(H	—	Km),
$PM_4 = H$	—	$PM_2 = H$	—	(H	1	Zh),	$PM_8 = H$	—	$PM_6 = H$	_	(H		Km).

Here the symbols - and | denote series and parallel connection of bodies, respectively. We consider some basic solutions of the differential equations for the behavior of composite bodies under load. The letter M denotes the elastic modulus of an H body.

Series and parallel connections of an H body with a Zh or Km body form bodies similar to Maxwell

and Kelvin (Voigt) bodies, where the viscous element is an N body. Here we do not differentiate between the viscosity and plastic flow, because the difference here is only in the form of the analytical relation between the flow rate and the stresses.

The rheological equation of a  $PM_1$  body is written as the equality of the rate of total strain to the sum of the elastic strain rate of the H body and the plastic strain rate of the Zh body (5):

$$\frac{1}{M}\frac{d\sigma}{dt} + A\exp\left(B\sigma\right) = \frac{d\varepsilon}{dt}.$$
(7)

The solutions of (7) are as follows

• at constant strain  $(d\varepsilon/dt = 0)$ , we have the stress relaxation equation

$$\sigma = -\frac{1}{B} \ln[\exp(-B\sigma_0) + ABMt]; \qquad (8)$$

• at constant strain rate  $d\varepsilon/dt = C$ , after replacing the integration constant in (8) by a function and obtaining a linear equation, we have the relation

$$\sigma = -\frac{1}{B} \ln\{\exp\left[-B(\sigma_0 + MCt)\right] + \frac{A}{C} \left[1 - \exp\left(-BMCt\right)\right]\};$$
(9)

which, as  $t \to \infty$ , gives the flow stress

$$\sigma = -\frac{1}{B} \ln\left(\frac{A}{C}\right),\tag{10}$$

which depends on the strain rate and the temperature;

• for loading at a constant rate  $d\sigma/dt = D$ , we obtain the following time dependence of the strain:

$$\varepsilon = \varepsilon_0 + \frac{Dt}{M} + A \exp\left(B\sigma_0\right) \frac{\exp\left(BDt\right) - 1}{BD}.$$
(11)

Here  $\sigma_0$  and  $\varepsilon_0$  are the stress and strain at t = 0.

At low stresses, at which the first term under the logarithm is much larger than the second, relation (9) gives a proportional dependence of stresses on time, and, hence, on strain. As the stresses increase and the terms become comparable, a smooth transition to constant flow stresses, given by formula (10), is observed. Under mild loading [solution (11)], the plastic strain increases exponentially with increase in the stresses, also giving the impression of the existence of a "yield limit."

A comparison of solutions (9) and (11) shows that the stress-strain relationship cannot be unambiguous and depends on the loading method. The solutions are equivalent only for small  $\sigma$ , for which plastic strains are negligible. With increase in the stresses, they give the natural transition from elastic to elastoplastic strains.

For a PM<sub>5</sub> body, the rheological equation is the same as (7) with replacement of (5) by (6):

$$\frac{1}{M}\frac{d\sigma}{dt} + 2A\sinh\left(B\sigma\right) = \frac{d\varepsilon}{dt}.$$
(12)

It has the following solutions.

For severe loading at  $d\varepsilon/dt = 0$ , integrating (12) gives

$$\sigma = \frac{2}{B} \operatorname{Artanh} \left( \tanh \left( \frac{B\sigma_0}{2} \right) \exp \left( -2ABMt \right) \right)$$
(13)

or, for calculation procedures,

$$\sigma = \frac{1}{B} \ln \frac{1+X}{1-X} \quad \left[ X = \frac{\exp\left(B\sigma_0\right) - 1}{\exp\left(B\sigma_0\right) + 1} \exp\left(-2ABMt\right) \right].$$

For  $d\varepsilon/dt = C$ , replacing the integration constant in (13) by a function, we arrive at the Riccati equation, which becomes a second-order linear differential equation [16]. Reverse transformation of its general integral gives the solution

$$\sigma = \frac{2}{B} \operatorname{Artanh} \left[ \frac{K_1 + K_2 K_3 \exp\left(-K_0 t\right)}{1 - K_3 \exp\left(-K_0 t\right)} \right],\tag{14}$$

where

$$K_0 = BM\sqrt{4A^2 + C^2}, \quad K_1 = (\sqrt{4A^2 + C^2} - 2A)/C,$$
  

$$K_2 = (\sqrt{4A^2 + C^2} + 2A)/C; \quad K_3 = \frac{\exp(B\sigma_0) - 1 - K_1[\exp(B\sigma_0) + 1]}{\exp(B\sigma_0) - 1 + K_2[\exp(B\sigma_0) + 1]}.$$

As  $t \to \infty$ , from (14) we obtain the "yield limit"

$$\sigma = \frac{2}{B} \operatorname{Artanh}(K_1). \tag{15}$$

For mild loading at constant rate D, we have

$$\varepsilon = \varepsilon_0 + \frac{Dt}{M} + \frac{2A}{BD} \left\{ \cosh \left[ B(\sigma_0 + Dt) \right] - \cosh \left( B\sigma_0 \right) \right\}.$$
(16)

At large stresses, solutions (8)-(11) and (13)-(16) give exactly the same results. Therefore, in computational procedures using models with Km bodies, it is reasonable to employ simpler models with a Zh body in appropriate regions of the loading program. This is also true for algorithms for processing experimental data in determining parameters of rheological models.

For a  $PM_2$  body, solving the equations of equilibrium and compatibility of strains, we obtain the rheological equation

$$\frac{d\varepsilon}{dt}\exp\left(BM\varepsilon\right) = A\exp\left(B\sigma\right).$$
(17)

Integrating this equation for  $\sigma = \sigma_0 + Dt$  leads to the solution

$$\varepsilon = \frac{1}{M} \left[ \sigma_0 + Dt + \frac{1}{B} \ln \left\{ \exp\left[ -B(\sigma_0 - M\varepsilon_0 + Dt) \right] + \frac{AM}{D} \left[ 1 - \exp\left( -BDt \right) \right] \right\} \right].$$
(18)

For a loading rate D = 0, we obtain

$$\varepsilon = \frac{1}{M} \left[ \sigma_0 + \frac{1}{B} \ln \left\{ \exp \left[ -B(\sigma_0 - M\varepsilon_0) \right] + ABMt \right\} \right].$$
(19)

For a PM<sub>6</sub> body, one obtains solutions of the rheological equation  $d\varepsilon/dt = 2A \sinh [B(\sigma - M\varepsilon)]$ immediately from (13) and (14) by reducing this equation to (12) and expressing  $\varepsilon$  in terms of stresses in a viscous element: for  $d\sigma/dt = D$ , we have

$$\varepsilon = \frac{1}{M} \bigg\{ \sigma_0 + Dt - \frac{2}{B} \operatorname{Artanh} \bigg[ \frac{K_1 + K_2 K_3 \exp(-K_0 t)}{1 - K_3 \exp(-K_0 t)} \bigg] \bigg\},$$
(20)

where

$$K_{0} = B\sqrt{4A^{2}M^{2} + D^{2}}, \quad K_{1} = (\sqrt{4A^{2}M^{2} + D^{2}} - 2AM)/D, \quad K_{2} = (\sqrt{4A^{2}M^{2} + D^{2}} + 2AM)/D,$$
$$K_{3} = \frac{\exp\left[B(\sigma_{0} - M\varepsilon_{0})\right] - 1 - K_{1}\left\{\exp\left[B(\sigma_{0} - M\varepsilon_{0})\right] + 1\right\}}{\exp\left[B(\sigma_{0} - M\varepsilon_{0})\right] - 1 + K_{2}\left\{\exp\left[B(\sigma_{0} - M\varepsilon_{0})\right] + 1\right\}}$$

and for D = 0, we have

$$\varepsilon = \frac{1}{M} \left[ \sigma_0 - \frac{2}{B} \operatorname{Artanh} \left\{ \tanh\left[\frac{B(\sigma_0 - M\varepsilon_0)}{2}\right] \exp\left(-2ABMt\right) \right\} \right].$$
(21)

If the material behaves the same in tension and compression, in going over to the compression region, only the signs of stresses, strains, and their rates should be altered. Otherwise, the parameters A and B must be different.

When the stresses in (18) are larger than  $-\ln(AM/D)/B$  or the stresses in (20) are larger than  $2 \operatorname{Artanh}(K_1)/B$ , a functional relationship between stresses and strains is seemingly observed under loading. In this case, we deal with "hardening plasticity." If the loading is stopped, we obtain equations of so-called logarithmic creep [17], which was treated analytically in (19) and (21). In processing experimental data of a real material, one should differentiate between plastic flow involving real hardening, where the structure

of the material changes and the activation volume  $\alpha$  decreases, and flow determined by various deformation mechanisms each of which have characteristic activation parameters.

Three-element models with an N body (a standard inelastic body) describe inelasticity of a relaxation type — amplitude-independent internal friction [18]. By virtue of the characteristic dependence of the flow rate on the stresses, replacement of an N body by a Zh or Km body leads to analytical dependences that describe inelasticity of a hysteresis type — amplitude-dependent internal friction. In metals and alloys, the latter is associated with dislocation hysteresis [18], which results from thermally activated motion of dislocations [19].

Solutions of the differential equations for  $PM_3$ ,  $PM_4$  and  $PM_7$ ,  $PM_8$  bodies are found by reducing these equations to the form (7) or (12) and using the solutions obtained above with the corresponding substitution of variables, parameters, and initial conditions. In analysis of internal friction, they allow one to calculate small inelastic strains when direct measurements are difficult and to describe the transition from severe loading of a solid body to mild loading and vice versa [13].

To reflect the properties of a real material, structural models [3, 13] consisting of a number of elements each of which characterizes one or another structural factor or strain mechanism are required. Solutions of multielement models are obtained by generalization of solutions of the rheological equations of threeelement models. Parameters of individual structural elements of the models can also be variable. For example, depending on the properties of the specific material, the creep element of a PM<sub>1</sub> body can have its internal structure as well [13], owing to which, attendant relaxation processes affecting the strain are modeled. Assuming that the model parameters are constant in the time step t, one can use solutions of the differential equations of ideal bodies to reproduce the deformation properties of real alloys.

Here, in essence, the general approach formulated by Rabotnov [20] is realized. In this approach, the creep rate is determined by the stress, the temperature, and a certain number of structural parameters. In this case, the structural parameters of the material itself can be determined quantitatively from the model parameters.

Use of the Rheological Models. Let us show some possibilities of the solutions obtained in modeling and analysis of the deformation properties of materials. We analyzed rather fully the deformation characteristics of D16 T aluminum alloy [13]. We give examples of calculations using a structural model of this alloy that describes hysteresis and creep.

As a test problem, we study creep with a stress jump. Experimental results of such tests have long been known [17]: in the unsteady stage of creep, the initial stress increased or decreased by a jump, and after some time it returned to the same level.

Figure 1 gives results of calculations of the deformation process in the D16 T alloy performed using its model. The flow characteristics of the material predicted by the model are exactly the same as those of the real flow. No additional conditions apart from specification of the loading program and the temperature are required.

This model of the material was used to estimate residual stresses and their relaxation in time on the contour of a hole in a thin-walled structural element. Figure 2 shows experimental and calculated residual stresses on the contour of a central hole in a sheet after single tensile loading and unloading followed by exposure to different temperatures. In experiments, the release method of [21] was used. Calculations were performed ignoring the structural instability of the alloy and can be considered quite satisfactory, because no relaxation tests were performed.

The inelasticity of a material under cyclic loading is usually characterized by the area and shape of the hysteresis loop. When rheological models with Zh or Km bodies are used, these characteristics are obtained directly by calculations by time steps with variation of the loading parameters. Figure 3 shows plastic-hysteresis loops calculated using the model of D16 T alloy for two forms of loading cycle. The model describes stabilization of the loop, the change in its shape, and accumulation of residual stresses. The damping characteristics of the material, which are required, for example, to determine the loading of structures under vibration, are calculated by the same method.

In all the above examples, the model predicts precisely the behavior of the material. The model parameters are obtained by processing only creep test results at constant stresses and strain diagrams of



Fig. 1. Creep with a stress jump: calculations using the model of the material D16T (423 K),  $\sigma_1 = 300$  MPa and  $\sigma_2 = 270$  and 310 MPa.

Fig. 2. Relaxation of residual stresses on the contour of a central hole of  $\emptyset$  6 mm in a sheet 80 mm wide and 3.5 mm thick after single loading at 293 K to a nominal stress of 300 MPa and relaxation: an experiment with 95% confidence intervals [21] (curves 1 and 2) and a calculation [13] (curves 1' and 2'); curves 1 and 1' refer to 293 K and curves 2 and 2' refer to 423 K (D16 T).



Fig. 3. Calculation of plastic-hysteresis loops for D16 T deformed by 0.2% (293 K,  $9 \cdot 10^{-3}$  Hz): (a) symmetrical-sawtooth-shaped cycle of loading; (b) sinusoidal cycle of loading.

Fig. 4. Experimental creep curves for 50KhFA steel at 643 K and activation volumes for the viscous element of a PM<sub>6</sub> body calculated from the creep curves: curves 1 and 2 refer to  $\sigma = 400$  and 500 MPa, and curves 3 and 4 refer to the activation volumes  $\alpha_2$  for curves 1 and 2.

the alloy under monotonic loading.

When experimental data are insufficient or the structure of the material changes during flow, analytical solutions of the rheological equations of physical media can be used. In structural transformations in alloys, the activation parameters of the deformation process change. The activation volume  $\alpha$  ranks first in the degree of effect of structural changes on the activation parameters of the flow process. Next follow the initial activation energy  $Q_0$  and the preexponent in (4) [10, 11, 13]. We give an example of analysis of the strain characteristics of a structurally unstable alloy and show the possibility of extracting maximum information with limited experimental data.

We analyze the initial period of creep at constant stresses using the curves in Fig. 4. Specimens of 50KhFA steel (0.45-0.55% C, 0.5-0.8% Mn, 0.17-0.37% Si, 0.8-0.1% Cr, and 0.1-0.2% V) were tested after quenching and low-temperature tempering.<sup>1</sup> It is known that exposure to stress leads the tempering process, called dynamic aging. As a result, the structure of the material changes [22]. This is evidenced by a decrease

<sup>&</sup>lt;sup>1</sup>The experimental part of this work was performed by S. A. Katarushkin, V. N. Chaplygin, and S. A. Laznenko.

in the strength of the steel. In two test regimes, specimens were subjected to stress for different times and unloaded, and reverse flow of the material was recorded (decrease in residual strain).

The rheological formula of a body that reproduces this plastic-flow process is of the form

$$PM_9 = Km_1 - PM_6 = Km_1 - (H_2 | Km_2),$$

where Km<sub>1</sub> describes the steady creep stage and PM<sub>6</sub> describes the initial creep period and creep recovery after relaxation. The elastic body H<sub>1</sub> was eliminated, and the elastic strain component was not considered. In tests performed on an MTS-10 machine, the loading and unloading times were recorded. This was also taken into account in analysis of the curves. The experimental errors of the relative strains were  $\pm 4 \cdot 10^{-5}$ .

We first analyze creep recovery. We use the simpler solution (19) for a PM<sub>2</sub> body, because the initial period of creep recovery is associated with large residual stresses in it. This solution corresponds to solution (21).

If, from the moment of unloading, the strain decreases by  $\Delta \varepsilon_1$  in time  $t_1$  and by  $\Delta \varepsilon_2$  in time  $t_2$ , using solution (19) we write the relation

$$\frac{t_2}{t_1} = \frac{\exp\left(B_2 M_2 \Delta \varepsilon_2\right) - 1}{\exp\left(B_2 M_2 \Delta \varepsilon_1\right) - 1},$$

from which the product  $B_2M_2$  is obtained. The difference between the values of this product for two curves of creep recovery was  $\pm 8\%$  of its average value. This is explainable by the difference in the properties of the tested specimens.

Then, using solution (21), for the initial creep period (a PM<sub>6</sub> body) we write the same relation for two time intervals from the initial reference point  $t_0$  with strain  $\varepsilon_0$  to the final times  $t_1$  and  $t_2$  with strains  $\varepsilon_1$  and  $\varepsilon_2$ :

$$\frac{t_2-t_0}{t_1-t_0} = \frac{\ln\left\{\tanh\left[B_2(\sigma_0 - M_2\varepsilon_2)\right]/\tanh\left[B_2(\sigma_0 - M_2\varepsilon_0)\right]\right\}}{\ln\left\{\tanh\left[B_2(\sigma_0 - M_2\varepsilon_1)\right]/\tanh\left[B_2(\sigma_0 - M_2\varepsilon_0)\right]\right\}}$$

Using this relation for a number of values of the modulus  $M_2$  we determine a number of values of  $B_2$ , subtracting the presumable contribution of the constant component of the creep rate from  $\varepsilon_0$ ,  $\varepsilon_1$ , and  $\varepsilon_2$ , so that the product  $B_2M_2$  corresponds to the stage of creep recovery. Then, for each pair of values of  $B_2$  and  $M_2$ , from (21) we find the parameter  $A_2$ . As a result, for both loading regimes we have two groups of the parameters  $A_2$ ,  $B_2$ , and  $M_2$ , from which the parameters that best describe the experimental creep curves should be chosen.

Construction of creep curves using the parameter values found shows that for constant values of them, the calculated creep curve does not agree with the experimental curve everywhere. With insufficient experimental data, it is natural to associate departures from the ideal-material model with the parameter that is most sensitive to structural changes, i.e., with the activation volume  $\alpha$ . Assuming that the parameters  $A_2$ and  $M_2$  are constant, we analyze the creep curves by time steps, determining the values of  $B_2$  (and, hence,  $\alpha_2$ ) in each time step provided that there is complete agreement (within the measurement error) between the calculated and experimental strains. In this case, from the groups of parameters for each loading regime, we choose single values of  $A_2$  and  $M_2$  such that, at the moment of unloading, the product  $B_2M_2$  corresponds to that obtained previously and the calculated creep recovery curve corresponds to the experimental curve.

Figure 4 shows the calculated activation volume  $\alpha_2$ . In the initial period of flow,  $\alpha_2$  grows rapidly (curves 3 and 4) to a practically constant value. This is supported by material hardness measurements. The hardness of a specimen tested at 500 MPa decreased by 3 units (initial hardness HRC<sub>ref</sub> = 44), and the hardness of a specimen tested at 400 MPa decreased by 3.5 units (initial hardness HRC<sub>ref</sub> = 42), i.e., by almost the same value, although the test time was severalfold longer.

The difference between the maximum values of the parameter  $\alpha_2$  in both test regimes is within the usual determination errors due to the difference in the properties of individual specimens [10]. In addition, here we do not take into account the possible change in the parameter  $A_2$ , which also increases somewhat in such situations [12]. This is evidenced by the somewhat larger residual strain at the neck of the specimens

after rupture tests at 643 K ( $\varepsilon_* = 0.75$ ) compared with tests at 293 K ( $\varepsilon_* = 0.46$ ). Of course, for quantitative estimates of changes in the remaining parameters, tests at other temperatures are required.

Although the behavior of  $\alpha_2$  is the same, it increases more rapidly at larger stresses. This is explained by a more rapid course of diffusion processes. The rate of the structural transformations due to these processes is expressed by a relation similar to (4) [23, 24]. The change in  $\alpha_2$  from the initial value  $\alpha_2^0$  to the largest value is satisfactorily described by the Avrami equation with exponent n > 4

$$\alpha_2 = \alpha_2^0 + \Delta \alpha_2 [1 + \exp\left(-kt^n\right)],$$

which is used in the formal theory of transformation kinetics [25]. The effective activation volume  $\beta$  leading to a change in the plasticity of the alloy can also be estimated. If, for  $\sigma_1$ ,  $\alpha_2$  increases to the largest value in time  $t_1$ , and for  $\sigma_2$ , it increases to the largest value in time  $t_2$ , we have  $\beta = RT \ln (t_1/t_2)/(\sigma_2 - \sigma_1) =$  $0.038 \text{ kJ/(mole} \cdot \text{MPa})$ . We compare this value with the value of the activation volume  $\alpha_1$ , which characterizes the constant component of the creep rate, in the same manner as was done in analysis of transformations of Duralumin [13]. Since in processing of the experimental creep curves, two velocity values have already been obtained, from (5) or (6) we calculate the parameters  $A_1$  and  $B_1$  and then  $\alpha_1$ . Here we see a similar pattern:  $\beta < \alpha_1$  [0.042 kJ/(mole  $\cdot$  MPa)]. This is explained by the lower average level of local stresses, which determine the rate of volume diffusion.

One can estimate the initial activation energies  $Q_0$  for deformation of Km<sub>1</sub> and Km<sub>2</sub> bodies, if, as a first approximation, the preexponent in (4) is set equal to  $10^{13}$  sec<sup>-1</sup>. The estimates obtained (257-287 kJ/mole) are close to the activation energy of self-diffusion in  $\alpha$ -Fe [26, 27].

The above examples show how large a body of information is provided by analysis of the rheological properties of materials and how many possibilities arise in modeling these properties if the mechanical models are filled with physical content. New information is extracted and explanations for the observed regularities in the behavior of solids are obtained. Going over from modeling of the macrocharacteristics of a deformable body to modeling of the associated processes not only extends the range of application of the models but also substantiates prediction where extrapolation of experimental dependences is required.

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